Expenditure versus consumption in the multi-good life cycle consumption model

Adang, P.J.M.

Publication date:
1991

Document Version
Publisher's PDF, also known as Version of record

TILBURG UNIVERSITY
KATHOLIEKE UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

EXPENDITURE VERSUS CONSUMPTION IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL

Pim Adang

FEW 517

EXPENDITURE VERSUS CONSUMPTION IN THE MULTI-GOOD LIFE CYCLE CONSUMPTION MODEL

Pim Adang
Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

October 1991.

The author wishes to thank Rob Alessie, Arie Kapteyn, Bertrand Melenberg and Arthur van Soest for stimulating discussions and helpful comments.

Abstract

In empirical studies on consumer behaviour, there often is a contrast between the theoretical and empirical sections. The models formulated in the theoretical part are concerned with consumption. The datasets used in the empirical part, however, rarely contain information on the actual consumption of consumers. Instead, they usually contain information on the purchases made by consumers. This distinction between consumption and expenditures may be important, and ignoring it could lead to incorrect inferences on the performance of the particular model under consideration.

Because of this potential risk, this paper is devoted to a study of this problem. The drawbacks of some well-known ways of dealing with it are discussed, and an alternative framework, assuming a life cycle context, is put forward. Next, the empirical applicability of this modified life cycle model is investigated. Attention is focussed especially on the situation which is most likely to occur, i.e., in which one has information on purchases, but not on consumption. It turns out that this lack of information seriously limits the applicability of the model.

1. Introduction.

In empirical studies on consumer behaviour, there often is a contrast between the theoretical and empirical sections. The models formulated in the theoretical part are concerned with consumption. The datasets used in the empirical part, however, rarely contain information on the actual consumption of consumers. Instead, they usually contain information on the purchases made by consumers. Depending on, among other factors, the extent to which the data are disaggregated into different commodity categories, and the length of the time interval used as reporting period, the distinction between consumption and expenditures may be important. Ignoring this distinction can lead to incorrect inferences on the performance of the model under consideration.

Because of this potential risk, some attempts have been made to take the aforementioned difference into account in the modelling phase. In the next section some approaches suggested in the literature are discussed. As they all suffer from some theoretical drawbacks, section 3 is devoted to the development of a model that tries to improve on these existing alternatives, assuming a life cycle context. In section 4, the empirical applicability of this modified life cycle model is investigated. Attention is focussed especially on the situation which is most likely to occur, i.e., in which one has information on purchases, but not on consumption. It turns out that this lack of information seriously limits the applicability of the model. More precisely, given this data limitation, the objective function of the modified model retains its (expected) utility function format, only if one assumes perfect foresight on the part of the consumer. Finally, some concluding remarks are made in section 5.

2. Existing solutions.

Several ways of modelling the difference between consumption and expenditures have been proposed in the literature. The best-known of these approaches probably is the one suggested in the so-called 'infrequency of purchase' literature, which emerged from studying consumption on a disaggregated level.

The starting-point for this model is often a demand equation for a certain good, where the fact that some persons are found not to consume the good, is usually dealt with by assuming a Tobit specification for this equation (cf., for example, Deaton and Irish (1984), Blundell and Meghir (1987) and Pudney (1989), section 4.4). Next, a link between (unobserved) consumption and (observed) expenditures is established by taking into account that what is bought during the reporting period, is not necessarily also consumed in the same period. This difference implies that the expenditure data are likely to differ from the underlying consumption pattern in the following two ways: firstly, the number of individuals reporting zero expenditures on a good will be larger than the number of individuals not consuming the commodity, and secondly, if the expenditures reported by individuals are positive, they will, on average, be larger than the corresponding consumption of these individuals. The infrequency of purchase approach tries to correct for these possible differences by adding an additional censoring process to the Tobit specification, thus scaling the positive expenditures downwards, and allowing for zero expenditures while the underlying consumption is positive. There are several objections which could be raised against this approach.

An important drawback of this approach is that the model is static. That is, although the infrequency of purchase model tries to establish a link between consumption and expenditures, it only links these quantities on a period by period basis. But because it is often possible that consumption in a certain period can be paid for not only in the period itself but also in periods preceding or following it, the link between consumption and expenditure should preferably be a multi-period one.[1] Ignoring this intertemporal aspect can easily result in incorrect inferences on the consumption pattern of consumers. To illustrate this, consider the following three-period example: assume someone consumes a certain good only in the second period, but divides the payment for it over all three periods. The infrequency of purchase models put forward in the literature will predict a consumption level for the second period which is at most equal to the purchases made in that period, and will, with positive probability, predict a positive consumption level in the other two periods. So, trying to model a dynamic process in a static framework can lead to serious distortions.

In order to take account of this problem, one could try to incorporate the infrequency of purchase approach in a dynamic model. In this study attention will be restricted to the probably best-known dynamic model: the life cycle model. Some of the consumption functions used in the static models can be interpreted as resulting from the second stage of a life cycle model in two-stage budgeting form (see, for instance, Meghir and Robin (1989)). So, a straightforward way of introducing dynamics could be to alter the life cycle model in such a way that the resulting consumption equations are no longer static, but also depend on past and future consumption. In this way one obtains a relation between consumption over time and thereby, after substituting the corresponding links between consumption and expenditures, in a relation between the expenditures in different periods. However, these links themselves remain static, implying that such a model still suffers from the aforementioned drawback.

Therefore, an alternative way of linking consumption and expenditures in a life cycle context will be proposed, which will be formalized in the next section. It can briefly be described as follows: the model retains the notion present in some infrequency of purchase studies, notably Meghir and Robin (1989), that individuals decide on their consumption and purchase strategies simultaneously. But in contrast with these studies, it takes account of this simultaneity by incorporating the link between consumption and expenditures from the outset in the life cycle model. Hence, the link is an integral part of the life cycle model itself, and no longer a separate model, employed only after the life cycle model has been solved.

The life cycle model which thus results has the following main characteristics: since the consumer derives utility from consuming a good and not from buying it, the utility function has consumption variables as its arguments. For the budget constraint, however, the opposite is true, that is, the price when buying a good is essential, not when consuming it. Therefore, the budget constraint depends on quantities boughtr, and not on quantities consumed. The link between the two is established by the fact that what is consumed in a certain period must be paid for sometime during the lifetime. This link implies, for instance, that aspects determining the expenditure pattern, like the timing of payments, can also influence consumption. So, the choice of the consumption path and the choice of the expenditure pattern determine, either directly or indirectly, the maximum expected utility which can be obtained by a consumer.

Apart from the aforementioned fact that the infrequency of purchase models suggested in the literature are static, they have another, less important, drawback. In many of these models it is assumed that the consumption and expenditure variables are normally distributed. This normality assumption could be a cause of misspecification. Since one of the advantages of the life cycle model is that it can be estimated without imposing such distributional assumptions, one would rather do without them.[2]

The remainder of this section is devoted to a discussion of two other ways of linking consumption and expenditures which are proposed in the literature, and which can be considered as special cases of the model to be introduced in the next section. The two links can be regarded as two different representations of one and the same model, which will be called the 'lag' model. It allows for an intertemporal link between consumption and expenditures, and is usually applied within a life cycle context. Starting-point for this lag model is the assumption that because of the durability of commodities, consumption in a certain period can be paid for in that period, or in previous periods. This is modelled by equating the consumption in a period to a function of the purchases made in that period and in earlier periods.

The first representation links current consumption to current and past expenditures by means of a lag polynomial. In some studies the polynomial is assumed to be finite (cf., for instance, Hansen and Singleton (1983), Hayashi (1985), Muelbauer (1988), Eichenbaum, Hansen and Singleton (1988) and Dunn and Singleton (1986)), whereas it is assumed infinite in other studies (see e.g. Neusser (1988) and Dunn and Singleton (1986) for durables).

The second, nowadays less commonly used representation, links consumption and expenditures by introducing a stock model. Purchases of a good lead to an increase of the available stock of this good, and the assumption that a constant fraction of this stock is consumed in every period establishes the link between consumption and expenditures.[3] Examples of this approach are the papers by Spinnewyn (1981), Pashardes (1986) and Bar-Ilan and Blinder (1988).

The equivalence of the two representations is easily demonstrated. Assuming a geometric decay structure, for example, the former one can be written as follows (where $N \leq \infty$):[4]

$$c_{t,i} = A(L)e_{t,i} \equiv \sum_{j=0}^{N} a_i^{j+1} \cdot e_{t-j,i}; \qquad 0 \leq a_i \leq 1 \tag{2.1}$$

where $c_{t,i}$ = period t's consumption of good i

$e_{\tau,i}$ = period τ's expenditutres on good i, $\tau = t-N,\ldots,t$

Under the same assumption regarding the decay structure, the second representation can be written as follows:

$$\begin{aligned} s_{t,i} &= \sum_{j=0}^{N} \vartheta_i^{j+1} \cdot e_{t-j,i}; \qquad 0 \leq \vartheta_i \leq 1 \\ c_{t,i} &= \alpha_i \cdot s_{t,i}; \qquad 0 \leq \alpha_i \leq 1 \end{aligned} \tag{2.2}$$

where $s_{t,i}$ = period t's stock of good i

By choosing a_i equal to $\alpha_i \cdot \vartheta_i$, the equivalence of both representations is established.

As stated before, the usual reason for introducing either one of these representations into a model constructed for explaining consumer behaviour, is to capture the difference between consumption and expenditures resulting from the durability aspect of some goods. However, if consumption is sufficiently disaggregated both over goods and over time,[5] there are also other reasons causing consumption to differ from expenditures. The most important one is the timing of the (registration of) payments. Some goods must be paid in advance (for example holiday reservations), whereas others can be paid after they have been consumed (for instance the telephone bill).[6] Moreover, even if the goods are paid during the period they are consumed, the payments need not be (completely) reported in this period. For example, it takes some time before payments made abroad are processed by banks and brought to one's attention. Or one could buy goods using a credit-card, which are charged only weeks later. If the reporting period is short, for example two weeks like in the often used British Family Expenditure Surveys, consumption and reported expenditures can differ, depending on what information is used by consumers when reporting their expenditures. Since these differences between the consumption and expenditure patterns are not taken into account by the lag model, it is less suited for modelling consumer behaviour on a disaggregated level.

Another reason for making the lag model less appropriate for modelling consumer behaviour at such a disaggregated level, is that it is difficult to account for zero consumption in this framework. This is because, as mentioned before, in many studies using this approach, it is assumed that the lag polynomial has an infinite length. This generally implies that once a purchase is made, the model "predicts" a positive consumption level (however small) in the period the purchase is made, and in all subsequent periods, thus resulting in a consumption path which is likely to be too smooth. Studies which do not assume an infinitely long lag polynomial, impose some maximimum lag (typically one or two periods) instead. However, since the reasons for choosing this maximimum lag are usually data driven rather than resulting from theoretical considerations, this is also not fully satisfactory. So, because zero consumption is likely to occur frequently if the consumption is sufficiently disaggregated, the lag model is less suited for handling such problems.

A less important disadvantage of the lag model is that it is difficult to combine with habit formation. Introducing habits in the lag polynomial representation requires incorporating an additional lag polynomial linking consumption over time.[7] However, the lag structure resulting from combining the two lag polynomials is by no means uniquely related to one particular combination of polynomials, as is illustrated by the following example.

Assume the following link between total consumption (c) and total expenditures (e):

$$c_t = e_t + a \cdot e_{t-1} \tag{2.3}$$

Furthermore assume that the negative effect of yesterday's consumption on utility derived from today's consumption, caused by habit formation, is as follows:

$$c^*_t = c_t - b \cdot c_{t-1} \tag{2.4}$$

Under the assumption that one has information on expenditures but not on consumption, one needs to combine both equations in order to obtain an expression for c^* in terms of the observed variables:

$$c^*_t = e_t + (a-b) \cdot e_{t-1} - a \cdot b e_{t-2} = e_t + \alpha \cdot e_{t-1} + \beta \cdot e_{t-2} \tag{2.5}$$

As can be seen from equation (2.5), the lag structure resulting from equations (2.3) and (2.4) can also be obtained by assuming, for example, no habit formation and a two-period lag for the link between consumption and expenditures, or assuming that expenditures equal consumption and that habits have an influence lasting two periods. The fact that such a one-to-one correspondence is lacking, implies that one cannot draw any clear-cut conclusions on the importance of habit formation on the one hand, and the link between consumption and expenditures on the other.

In the studies using the stock representation it is usually possible to draw conclusions on the role which each aspect plays. For instance, if one assumes, like Pashardes (1986), that the link between consumption and stock is linear and constant over time, and that the maximum lag (N in equation (2.2)) is infinite, the sign of the parameter ϑ_i in equation (2.2) determines whether the habit forming aspect outweighs the durability aspect. However, if not all of these assumptions are satisfied, this needs no longer hold true. If, for example, the link between consumption and stock is nonlinear, such an unambiguous interpretation of the parameter ϑ_i probably will not be possible.

Especially the assumption that a constant fraction of the stock is consumed in each period is troublesome, since the size of the stock is partly determined by market factors (like prices, and the minimum quantity of a good one must buy), whereas consumption is mainly determined by preferences. Assuming that a constant fraction of the stock is consumed implies, for example, that if a price cut in a certain period induces a consumer to buy a large quantity of a good to take advantage of this discount, his or her consumption *must* increase significantly in this period. Since there is no compelling reason why consumers should behave so rigidly, alternative consumption patterns could be just as plausible. One such alternative could be a pattern implying a constant consumption level as long as the available stock allows for it; so $c_t = \min[\bar{c}, s_t]$. Such a pattern perhaps could be a reasonable representation of the consumption of a commodity like clothing.

Given the aforementioned shortcomings, the lag model is not considered fully suited for establishing a link between consumption and expenditures, when studying individual consumer behaviour on a disaggregated level. The next section is, therefore, devoted to the development of an alternative framework which tries to improve upon the alternatives discussed in this section.

3. Linking consumption and expenditures in the life cycle model.

The models discussed in the previous section can be considered as belonging to either one of two different classes of life cycle models. The classes differ in the way in which they take account of the distinction between expenditures and consumption. The first class contains the infrequency of purchase models which can, under appropriate assumptions, be interpreted as a framework in which one first solves a life cycle model formulated in consumption terms only. In a second step, the thus resulting consumption equations are related to some expenditure variable.

In contrast, the second class of life cycle models, to which the lag model belongs, is characterized by the fact that the difference between consumption and expenditures is taking into account in the life cycle model itself. That is, the link between the consumption and expenditure variables is introduced by adding equality constraints to the model which link each period's consumption to some function of realized purchases. Because of this exact relationship between the consumption and purchase levels, choosing either of them, fully determines the other.

In this section a generalization of the models belonging to this second class is put forward. The key notion underlying these models- that utility is derived from consumption, and costs result from purchases- is retained. Hence, the utility function depends on consumption variables, and the budget constraint is determined by expenditure variables. The difference between the proposed model and the lag model is the way in which consumption and expenditures are linked to one another.

It is no longer assumed that the consumption in a certain period is exactly equal to a weighted sum of purchases realized until that period. Instead, it is assumed that expenditures imply an upper bound on the consumption of the different goods. If a particular good is a durable, the corresponding upper bound will depend on past purchases and past consumption. If one can postpone the payment of the consumption of a good, the corresponding upper bound will depend on the purchases which will be made in future periods.[8]

An advantage of the model proposed in this section, as compared with the lag model, is that it allows for a greater flexibility. For example, a price discount in a certain period might induce a consumer to buy a large quantity of the particular good in that period, without increasing his consumption of the good.[9] Or one might buy a durable good, for instance a car, whilst keeping one's consumption of the good unchanged. Because of the assumed equality between consumption and a weighted sum of realized purchases, both cases are not easily modelled using the lag model. In the framework proposed in this section, however, they can be modelled without great difficulty, since both examples simply lead to higher upper bounds for the particular goods. Thus higher consumption levels of these goods are possible, but by no means necessary.

A formal representation of the life cycle model in which this generalization is incorporated could be as follows (for $t = 1,\dots,L$):

$$\begin{aligned} &\operatorname*{Max}_{c_t, e_t, \dots c_L, e_L} \quad E_t \textstyle\sum_{\tau=t}^{L} \left(\frac{1}{1+\rho}\right)^{\tau-t} u(c_\tau) \\ &\text{s.t.} \ \textstyle\sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} p_\tau' e_\tau \ \le (1+r)A_{t-1} + \textstyle\sum_{\tau=t}^{L} \left(\frac{1}{1+r}\right)^{\tau-t} i_\tau, \\ &\qquad c_\tau \ge 0 \quad \tau = 1,\dots, L, \\ &\qquad c_1 \le \delta_{1,1} e_1 + \textstyle\sum_{s=2}^{L} \alpha_{s,2} e_s, \\ &\qquad c_\tau \le \delta_{\tau,\tau} e_\tau + \textstyle\sum_{s=1}^{\tau-1} \delta_{s,\tau}(e_s - c_s) + \textstyle\sum_{s=\tau+1}^{L} \alpha_{s,\tau} e_s \quad \tau = 2,\dots, L-1, \end{aligned} \tag{3.1}$$

$$c_L \leq \delta_{L,L}e_L + \Sigma_{s=1}^{L-1}\delta_{s,L}(e_s-c_s).$$

where

$u(\cdot)$ = within period utility function; assumed to be strictly concave, constant over time and increasing in its arguments,

c_τ = $(c_{\tau,1},\ldots,c_{\tau,M})'$: M-dimensional vector of *consumption* of goods in period τ,

e_τ = $(e_{\tau,1},\ldots,e_{\tau,M})'$: M-dimensional vector of *purchases* of goods in period τ,

p_τ = $(p_{\tau,1},\ldots,p_{\tau,M})'$: M-dimensional price vector of the goods in period τ,

i_τ = nominal non-property income in period τ,

r = nominal interest rate; assumed to be constant over time,

ρ = time preference parameter,

A_{t-1} = assets available at the beginning of period t,

E_t = expectation conditional on the information available at period t,

$\alpha_{s,\tau}$ = $\text{diag}(\alpha_{s,\tau,1},\ldots,\ \alpha_{s,\tau,M})$; (M×M) diagonal matrix with as diagonal elements the fractions of period s' expenditures on each good which can be consumed in period $\tau \leq s$,

$$\begin{aligned} &\alpha_{s,\tau,i} \in [0,1] \\ &\qquad\qquad i = 1,..,M; \quad \tau = 1,..,L; \quad s = \tau+1,..,L \\ &\alpha_{s,\tau+1,i} \geq \alpha_{s,\tau,i} \end{aligned} \tag{3.2}$$

$\delta_{s,\tau}$ = diag($\delta_{s,\tau,1},\ldots,\ \delta_{s,\tau,M}$); (M×M) diagonal matrix with on the diagonal the fractions of not consumed outlays on each good done in period s, which are available in period $\tau \geq s$,

$$\delta_{1,\tau,i} = 1 \text{ if } c_{1,i} \geq e_{1,i}$$
$$i = 1,\ldots,M;\ \tau = 1,\ldots,L$$
$$\delta_{1,\tau,i} \in [0,1] \text{ if } c_{1,i} < e_{1,i}$$

$$\delta_{s,\tau,i} = 1 \text{ if } c_{s,i} \geq e_{s,i} + \Sigma_{l=1}^{s-1}\delta_{l,s,i}(e_{l,i} - c_{l,i})$$
$$i = 1,\ldots,M;\ s = 2,\ldots,L;\ \tau = s,\ldots,L$$
$$\delta_{s,\tau,i} \in [0,1] \text{ if } c_{s,i} < e_{s,i} + \Sigma_{l=1}^{s-1}\delta_{l,s,i}(e_{l,i} - c_{l,i}).$$

The model put forward in (3.1) can be seen as a modification of the multi-good version of Hall's (1978) life cycle consumption model under uncertainty. As can be seen from the above formulation, a consequence of loosening the tie between consumption and expenditures is that in order to achieve the maximum expected lifetime utility, the model must be solved with respect to the consumption as well as the expenditure variables. This contrasts with the lag model, in which the link between expenditures and consumption implies that the models need to be maximized only with respect to either one of these variables. As stated before, the difference between consumption and expenditures implies a budget constraint which depends on expenditure variables. The other constraints in (3.1) provide the link between consumption and purchases.[10] Their specific form is determined by the aspects mentioned earlier: durability and postponement.

If a good is durable, a certain fraction of the quantity bought in a period will, by definition, be available in the next period(s). This aspect is represented by the part of the right hand side of these constraints relating to past expenditures. As can be seen from the way in which this is modelled, both the consumption in the periods prior to the particular period under consideration, as well as technical factors influencing the rate of decay ($\delta_{s,\tau}$), determine the exact quantity which is available in future periods.

Notice that in lag models these two elements are usually not separated. Especially the effect of consuming on the stock available next period is ignored, as depreciation is considered to be the only reason for a decrease in the available stock.

The second aspect determining the link between consumption and expenditures is the postponement of payments. This implies that a certain quantity of a good can be consumed in one period, and only be paid for in later periods. This aspect is represented by the part of the right hand side of the 'linking' constraints relating to purchases in periods succeeding the particular period under consideration. The assumption that payments which can be delayed $s-\tau$ periods can also be postponed one period less, implies the restriction on the $\alpha_{s,\tau}$'s given in (3.2).

Another effect worth pointing out is that incorporating the postponement aspect complicates the way in which the durability aspect is modelled. The possibility of delaying the payment implies that one has to determine for each good in each period, starting with period t, whether the corresponding consumption level exceeds the quantity remaining from the purchases made until that moment. If this is the case, a certain quantity has to be paid for in later periods, hence the durability parameter corresponding to this good and period, i.e., $\delta_{s,\tau,i}$, is set equal to one, since future payment obligations do not decrease over time. If the consumption level does not exceed the available quantity, the durability parameter takes a value between zero and one, depending on technical factors influencing the rate of decay.

Finally, notice that nonnegativity constraints are imposed on the consumption variables only. Expenditures can become negative, since one can sell (a part of) the stock one has built up in previous periods.[11] So the lower bound for the purchases of a good in a certain period is minus the quantity available at the beginning of this period.

In the remainder of this section it will be established that the life cycle model without the difference between consumption and expenditures, the lag model, and the life cycle model with habit formation, all are special cases of the model given in (3.1). The common features of these models are that they do not allow for the postponement of payments, and that the restrictions linking consumption to expenditures are equality constraints. This implies that in these constraints the $\alpha_{s,\tau}$'s are set equal to zero, and that the inequality signs are replaced by equality signs.

The additional requirement needed in order to obtain the traditional life cycle model in which the difference between consumption and expenditures is not taken into account, is to set all $\delta_{s,\tau}$'s equal to zero. The lag model results if one imposes a reparameterization on the $\delta_{s,\tau}$'s. The stock representation as given in (2.2), for instance, results if one substitutes the consumption realized until period τ in period τ's constraint, and makes use of the following reparameterization:

$$\delta_{s,\tau,i} = \alpha_i \vartheta_i \left(\frac{\vartheta_i}{1-\alpha_i \vartheta_i}\right)^{\tau-s} \qquad i = 1,..,M;\ s = 1,..,L;\ \tau = s,..,L$$

Similarly, the polynomial representation given in (2.1) results after substituting in each period's constraint the consumption realized until that period, and using the following reparameterization:

$$\delta_{s,\tau,i} = a_i \left(\frac{a_i}{1-a_i}\right)^{\tau-s} \qquad i = 1,..,M;\ s = 1,..,L;\ \tau = s,..,L$$

Since habit formation is modelled by introducing a similar polynomial, as was already pointed out in section 2, the life cycle model with habit formation can be obtained using the same procedure.

So it can be concluded that a number of well-known models are special cases of the life cycle model introduced in this section. In the next section it is determined under what conditions this life cycle model can be used in empirical applications.

4. Empirical applicability.

The life cycle model as formulated in (3.1) can be estimated if one has information on both consumption and expenditures. However, as already mentioned before, it is very rare to find a dataset containing information on consumption.[12] In most datasets one only finds information regarding the purchase of commodities. Hence, the question which is addressed first, is what conditions must be imposed to enable the estimation of the model given in (3.1) using expenditure data only. In subsection 4.1, this question is taken up for the model under uncertainty, whereas the life cycle model assuming perfect foresight is considered in subsection 4.2. In subsection 4.3 the conclusions for the life cycle model under uncertainty arrived at in subsection 4.1 are compared with those which can be drawn if one has consumption data at one's disposal.

4.1 The life cycle model under uncertainty.

The usual way of estimating models like the one given in (3.1) is to combine the first order conditions into a system of equations which can be estimated on the basis of the data available. In order to derive the first order conditions for the model given in (3.1), a generalized Lagrange multiplier rule, whose working in a life cycle context is discussed by Melenberg and Alessie (1989), is applied. This results in the following system of restrictions:

$$E_t[\ \sum_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\sum_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau' h_\tau^e + \sum_{\tau=1}^{L}\mu_\tau' h_\tau^c +$$

$$\sum_{\tau=1}^{L}\sum_{i=1}^{M}\nu_{\tau,i}\cdot[D_e R_{\tau,i}' h_i^e - D_c R_{\tau,i}' h_i^c]\ ] = 0 \qquad (4.1)$$

such that

$$E_t\ [\mu_{\tau i}\cdot c_{\tau i}] = 0, \qquad i = 1,\dots, M;\ \tau = 1,\dots, L$$
$$E_t\ [\nu_{\tau i}\cdot R_{\tau i}] = 0.$$

where

$D_c u(c_\tau) = (\frac{\partial u(c_\tau)}{\partial c_{\tau,1}},\dots,\frac{\partial u(c_\tau)}{\partial c_{\tau,M}})'$: vector of partial derivatives of $u(\cdot)$ with respect to the consumption variables,

$h_\tau^c = (h_{\tau,1}^c,\dots, h_{\tau,M}^c)'$: vector of functions were $h_{\tau,i}^c$ is allowed to depend on all variables influencing $c_{\tau,i}$ (cf. section 3 in chapter 2),

h^e_τ = $(h^e_{\tau,1},\ldots, h^e_{\tau,M})'$: vector of functions were $h^e_{\tau,i}$ is allowed to depend on all variables influencing $e_{\tau,i}$,

h^c_i = $(h^c_{1,i},\ldots, h^c_{L,i})'$,

h^e_i = $(h^e_{1,i},\ldots, h^e_{L,i})'$,

λ_t = Lagrange multiplier corresponding with the lifetime budget constraint,

μ_τ = $(\mu_{\tau,1},\ldots, \mu_{\tau,M})'$: vector of Lagrange multipliers correpsonding with the nonnegativity constraints for period τ's consumption,

$\nu_{\tau,i}$ = Lagrange multiplier corresponding with the upper bound on period τ's consumption of good i,

$R_{\tau,i}$ = the upper bound on period τ's consumption of good i with $c_{\tau,i}$ substracted from both sides of the inequality,[13]

$D_eR_{\tau,i}$ = $(\delta_{1,\tau,i},\ldots, \delta_{\tau,\tau,i}, \alpha_{\tau+1,\tau,i},\ldots, \alpha_{L,\tau,i})'$: $R_{\tau,i}$ differentiated with respect to the ith expenditure variable of each period, $\tau < L$,

$D_eR_{L,i}$ = $(\delta_{1,L,i},\ldots, \delta_{L,L,i})'$,

$D_cR_{\tau,i}$ = $(-\delta_{1,\tau,i},\ldots, -\delta_{\tau-1,\tau,i}, -1, 0,\ldots, 0)'$: $R_{\tau,i}$ differentiated with respect to the ith consumption variable of each period, $\tau > 1$,

$D_cR_{1,i}$ = $(-1, 0,\ldots, 0)'$.

In order to be able to estimate the first order conditions formulated above in the absence of consumption data, the parts depending on consumption variables must be eliminated. This is achieved by setting all $h^c_{\tau,i}$'s equal to zero. After this choice of the $h^c_{\tau,i}$'s is substituted in (4.1), the following condition results:

$$E_t[-\lambda_t \cdot \sum_{\tau=t}^{L} (\frac{1}{1+r})^{\tau-t} p'_\tau h^e_\tau + \sum_{\tau=1}^{L} \sum_{i=1}^{M} \nu_{\tau,i} \cdot D_e R'_{\tau,i} h^e_i] = 0 \qquad (4.2)$$

As (4.2) demonstrates, this procedure does not allow for the estimation of the parameters of the utility function. Nor is it straightforward to estimate the parameters characterizing the link between consumption and expenditures, i.e., the $\alpha_{s,\tau,i}$'s and the $\delta_{s,\tau,i}$'s. This is because the $h^e_{\tau,i}$'s can not be chosen in a way which eliminates the unknown Lagrange multipliers present in (4.2). Hence, estimating the parameters of the link between consumption and expenditures on the basis of (4.2), requires additional assumptions regarding these Lagrange multipliers.

Because of the difficulties with which one is confronted if one tries to estimate the model following the procedure desribed above, it could be worthwhile to consider an alternative approach for estimating model (3.1) on the basis of expenditure data only. This alternative consists of solving the model in two steps. In the first step, the model is maximized with respect to the consumption variables. Next, the optimal consumption bundle is written as a (vector-)function of the expenditure variables. After replacing the consumption variables by this function, a model which only depends on the expenditure variables results. In the second step this model is solved with respect to the expenditure variables. This second step model can then be used in estimation.

In order to study the working of this two step approach in greater detail, consider the following life cycle model which includes model (3.1) as a special case:[14]

$$\begin{aligned} &\max_{c,e} \quad U(c) \\ &\text{s.t. } \Phi(e,c) \in Z \qquad\qquad (4.3) \\ &\qquad (c,e) \in C \times E \end{aligned}$$

where

$U(\cdot)$ = expected lifetime utility function,

$C, E \subset L$,

L = set of functions with domain V and range $\mathbb{R}^p$,

V = set of possible values of the vector of uncertainty inducing variables v (the so-called input variables),

c = p-dimensional vector containing consumption functions for each good in each period,

e = p-dimensional vector containing the expenditure functions for each good in each period,

$\Phi(\cdot)$ = q-dimensional vector of constraints on c and e,

Z = $\{z(\cdot) \in L;\ z(\cdot) \geq 0\} \subset L$.

After replacing the consumption variables in (4.3) by functions of expenditure variables the following second step model results:

$$\begin{aligned} &\max_{e} \quad U(F(e)) \\ &\text{s.t. } \Phi(e,F(e)) \in Z \\ &\qquad (F(e),e) \in C\times E \end{aligned} \tag{4.4}$$

where F: $E \to C$, the (vector-)function relating the consumption variables to the expenditure variables; this function results from solving the first step (see the appendix for conditions under which the existence of this function is guaranteed).

An issue of particular interest is in what way this second step model is related to the models which are usually estimated, i.e., life cycle models which are (explicitly or implicitly) formulated in expenditure terms. Starting-point for the answer to this question is the model as given in (4.3), and in particular its objective function. Since this is an expected utility function, it can be written as follows:[15]

$$U(c) = \int_V u(c(v))\ dP(v) \tag{4.5}$$

where $u(\cdot)$: $\mathbb{R}^p \to \mathbb{R}$,

P = the probability distribution with respect to $v \in V$.

The objective function of the second step model results after replacing c by F(e)

$$U(F(e)) = \int_V u(F(e)(v))\ dP(v) \tag{4.6}$$

This function can be considered as an expected utility function if it can be written as follows:

$$U(F(e)) = \int_V \tilde{u}((e(v))\ dP(v) \tag{4.7}$$

for some $\tilde{u}$: $\mathbb{R}^p \to \mathbb{R}$

However, the function $u(F(e)(\cdot))$ whose expectation is determined in (4.6) can, in general, not be equal to the function $\tilde{u}(e(\cdot))$ in (4.7), as it depends on the complete function $e(\cdot)$, and not only on just one possible value of this function, say $e(v)$. Were this the case one could rewrite (4.6), using:

$$F(e)(v) = \tilde{F}(e(v))$$

for some $\tilde{F} \neq F$

Which after substituting would result in

$$u(F(e)(v) = u(F(e(v))) = \tilde{u}(e(v))$$

where $\tilde{u} = u \circ F$

To illustrate that this procedure will, in general, not be applicable, consider the example depicted in Figure 1 where it is assumed that there is just one uncertainty inducing variable (so v is a scalair):

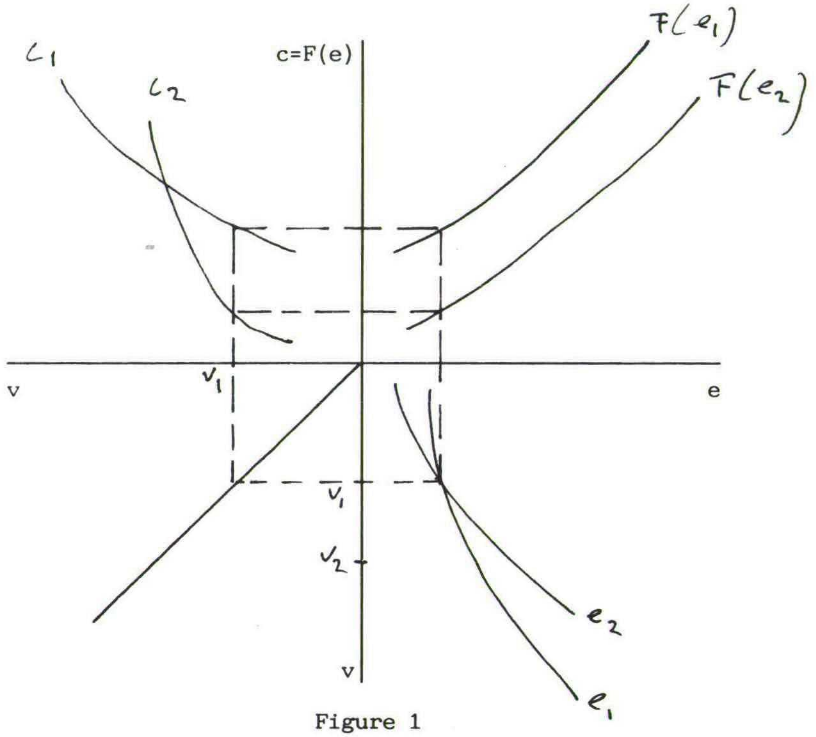

Figure 1

The functions $e_1(\cdot)$ and $e_2(\cdot)$, depicted in the fourth quadrant, are elements of E, whereas $c_1(\cdot)$ and $c_2(\cdot)$, drawn in the second quadrant, belong to the set C. In the first quadrant, the function which results from solving the first step, relating consumption to expenditures, is drawn. It is assumed this function $F(\cdot)$ links $e_1(\cdot)$ to $c_1(\cdot)$, and $e_2(\cdot)$ to $c_2(\cdot)$. In the third quadrant, the 45° line is drawn. As can be seen from this figure, it is not sufficient to know the value of the expenditure functions at v_1 in order to be able to determine whether $c_1(v_1)$ $(=F(e_1(v_1)))$ or $c_2(v_1)$ $(=F(e_2(v_1)))$ corresponds with this value. For this, at least one additional value of the functions $e_1(\cdot)$ and $e_2(\cdot)$ is required, for example, those corresponding with v_2.

From the above it can be inferred that if one starts out from a life cycle model as given in (4.3), the model one ends up with in the second step will, in general, not be the familiar life cycle model which has an expected utility function as its objective function. So, if one believes that a model like the one given in (4.3) is an adequate description of the consumer's optimization problem, one cannot draw any conclusions regarding this model on the basis of results obtained from estimating a life cycle model in expenditure terms, in which an expected utility function is used as objective function. Instead one should use a model like the one given in (4.4). However, since solving this model requires knowledge of the shape of all the $e_j(\cdot)$'s $\in$ E, this does not seem to be a straightforward task.

In summary, it can be concluded that both ways of estimating model (3.1) considered in this subsection, i.e., either directly from the first order conditions, or by using a two step approach, are applicable with great difficulty only, in the absence of consumption data.

4.2 The life cycle model under perfect foresight.

The problem with the two step approach discussed in the previous subsection was the fact that one needed to know all complete expenditure functions in order to be able to estimate the second step model. In case of the the life cycle model under perfect foresight, however, it is assumed that the consumer knows exactly which of the possible values of the vector of input variables v is realized. Hence, one only needs information on the value of the expenditure functions for this particular value of the input variables.

Given this feature, and the discussion in subsection 4.1, it is easily established that the objective function of the second stage model which results under the assumption of perfect foresight, is an (expected) utility function, implying that the second step model can be solved in the usual way. To demonstrate this, suppose that v_i is the actual realization of the input variables. The consumption and expenditure levels which are possible given this value v_i are $c_j(v_i)$ $\forall$ $c_j(\cdot) \in$ C, and $e_j(v_i)$ $\forall$ $e_j(\cdot) \in$ E, respectively. Since all other possible values of the consumption and expenditure functions are irrelevant for the consumer's optimization problem, as they will not occur, it is no longer necessary to know the complete consumption and expenditure functions. So, substituting F(e) for c in the (expected) utility function U(c) as defined in (4.5) now results in:

$$U(c) = u(c(v_i)) = u(F(e(v_i))(v_i)) = u(\tilde{F}(e(v_i))) = \tilde{u}(e(v_i)) \quad (4.8)$$

The function $\tilde{u}(\cdot)$ can be taken as an expected utility function, with a degenerated probability distibution of v, which concentrates all probability mass in the point v_i. Assuming v consists of just one variable, the consequences of the perfect foresight assumption can be illustrated by Figure 1. Suppose that v_1 is the value of the input variable which is realized. As Figure 1 shows, the corresponding values of the two expenditure functions are just a single point, and the two consumption functions collapse to two points. Hence, the expenditure level is fully determined, and of the two possible consumption levels, the one resulting in the highest utility level is chosen. In order to make this choice, one needs, in contrast with the model under uncertainty, no information on the values of $c_1(\cdot)$ and $c_2(\cdot)$ for other possible realizations of v.

So, if one is willing to assume perfect foresight on behalf of the consumers, estimating a life cycle model in expenditure terms with an (expected) utility function as its objective function, can give some insight in the original model as given in (4.3). However, because of the following two reasons it is doubtful whether one can learn very much from these estimation results.

Firstly, it will in general be difficult to identify the parameters of the original model from the (reduced form) parameters of the second step model. Secondly, any model resulting in the second step cannot be distinguished from a properly chosen life cycle model in which the distinction between consumption and expenditures is ignored, and which is from the outset formulated in expenditure terms only.

4.3 The life cycle model under uncertainty in the presence of consumption data.

Given information on consumption, estimation of model (3.1) does not seem too difficult a task. In order to get some insight in how to derive a system of restrictions from condition (4.1) which can be used for estimation, three special cases of model (3.1) will be considered in this subsection.

i) Only the lifetime budget constraint is binding:

Under this assumption, condition (4.1) collapses to the following restriction:

$$E_t[\ \Sigma_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\Sigma_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau' h_\tau^e] = 0 \qquad (4.9)$$

On the basis of (4.9), one can derive many different sets of moment restrictions which can be used in estimation. For example, by choosing $h_{t,1}^c = 1/p_{t,1}$, $h_{t+1,1}^c = -(1+r)/p_{t+1,1}$, and all other $h_{\tau,i}^*$'s (where * equals c or e) equal to zero, the Euler equation for the first good corresponding with the multi-good version of Hall's (1978) model results. Alternatively, if one has no information on the prices with which consumers are confronted[16], condition (4.9) still allows one, in contrast with the first order conditions of the multi-good version of Hall's (1978) model, to estimate the model, by setting, for instance, $h_{t,1}^c = 1$, and $h_{t+1,1}^c = -1$.

ii) Both the lifetime budget constraint and the nonnegativity constraints are binding:

In this case, condition (4.1) is reduced to the following restriction:

$$E_t[\ \Sigma_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h_\tau^c - \lambda_t\cdot\Sigma_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p_\tau' h_\tau^e + \Sigma_{\tau=1}^{L}\mu_\tau' h_\tau^c] = 0 \qquad (4.10)$$

Under the assumption that the nonnegativity constraints for the first good are not binding, and by a proper choice of the $h_{\tau,i}^*$'s, one can obtain the moment restrictions for the multi-good version of Hall's (1978) model with binding nonnegativity constraints. Choose, for instance, $h_{t,1}^c =$

1, $h^c_{t,2} = -I_{(0,\infty)}(c_{t,2})$, and $h^c_{t+1,1} = -(1+r)\cdot(p_{t,1} - p_{t,2}\cdot I_{(0,\infty)}(c_{t,2}))/p_{t+1,1}$, and set all other $h^*_{\tau,i}$'s equal to zero.

ii) Both the lifetime budget constraint and the upper bounds are binding:

The for this case relevant part of condition (4.1) is:

$$E_t[\ \sum_{\tau=t}^{L}(\frac{1}{1+\rho})^{\tau-t}D_c u(c_\tau)'h^c_\tau - \lambda_t\cdot\sum_{\tau=t}^{L}(\frac{1}{1+r})^{\tau-t}p'_\tau h^e_\tau +$$
$$\sum_{\tau=1}^{L}\sum_{i=1}^{M}\nu_{\tau,i}\cdot[D_e R'_{\tau,i}h^e_i - D_c R'_{\tau,i}h^c_i]\ ] = 0 \qquad (4.11)$$

To illustrate this case, set all $h^*_{\tau,i}$'s equal to zero, except $h^c_{t,1}$ and $h^c_{t+1,1}$. After substituting these zeroes and the values of $D_e R_{\tau,1}$ and $D_c R_{\tau,1}$, the following restriction results:

$$E_t[\frac{\partial u(c_t)}{\partial c_{t,1}}\cdot h^c_{t,1} + (\frac{1}{1+\rho})\cdot\frac{\partial u(c_{t+1})}{\partial c_{t+1,1}}\cdot h^c_{t+1,1} + \nu_{t,1}h^c_{t,1} +$$
$$\nu_{t+1,1}(\delta_{t,t+1,1}h^c_{t,1} + h^c_{t+1,1}) + \ldots\ldots +$$
$$\nu_{L,1}(\delta_{t,L,1}h^c_{t,1} + \delta_{t+1,L,1}h^c_{t+1,1}] = 0 \qquad (4.12)$$

By making some additional assumptions, this condition can be greatly simplified. For instance, under the assumption that the upper bound for period t is not binding, and that $\delta_{\tau,s,1} = \delta_1^{s-\tau}$ (i.e., a geometric decay structure), and by setting $h^c_{t+1,1} = -\delta_1\cdot h^c_{t,1}$, all unknown Lagrange multipliers drop out and the following condition results:

$$E_t\{[\frac{\partial u(c_t)}{\partial c_{t,1}} - (\frac{\delta_1}{1+\rho})\cdot\frac{\partial u(c_{t+1})}{\partial c_{t+1,1}}]\cdot h^c_{t,1}\} = 0 \qquad (4.13)$$

The special cases considered above indicate that, given information on consumption, a model like the one formulated in (3.1) can be estimated.[17] However, notice that the systems of restrictions derived as examples of these special cases, are by no means exhaustive. That is, conditions (4.9), (4.10), and (4.11) allow for many more (linearly independent) restrictions than those used in the examples (for instance, those in which not all $h^{e}_{\tau,i}$'s are set equal to zero). This is of importance, since estimating such a subsystem is inefficient and might make it impossible to estimate all parameters of interest. Furthermore, tests based on a subsystem of restrictions could be misleading.[18] This should be taken into account if one is in a situation allowing for the estimation of models of the type discussed here.

5. Concluding remarks.

In this chapter, the consequences for the life cycle model of taking into account the difference between expenditures and consumption were considered. Existing ways of dealing with this difference, i.e., the approaches used in infrequency of purchase models and in lag models, were discussed. As both types of models have their disadvantages, an alternative way of incorporating this difference in life cycle models was proposed.

Although this generalization seems attractive, the data requirements associated with it make it less suited for empirical applications. Put more precisely, the estimation of the model is straightforward only if one has information on the expenditures of households as well as on their consumption. Since information on the consumption is very rarely available, one is forced to express the model in expenditure terms only in order to enable estimation.

Two approaches for achieving this aim were considered in this chapter. The first one starts from the first order conditions of the model in which the link between consumption and expenditures is incorporated, and by proper combining of these conditions, removes all parts depending om consumption variables. However, it turned out that this procedure makes it impossible to estimate the parameters characterizing the utility function.

Therefore, a second approach was considered in which the model was solved in two steps. In the first step one solves the model with respect to the consumption variables. After expressing the solution of this first step model in expenditure terms, a model depending on expenditure variables only results. This can then be solved and estimated in a second step. It was demonstrated that only if one assumes perfect foresight, is this second step related to the type of life cycle model which is usually estimated, i.e., a model formulated in expenditure terms only which has an (expected) utility function as its objective function.

But even if one restricts one's attention to the life cycle model under perfect foresight, one is still confronted with an important problem. That is, the second step model which results under this assumption can not be distinguished from a properly chosen life cycle model in which the difference between consumption and expenditures is ignored, and which is from the outset formulated in expenditure terms only.

The overall conclusion which can be drawn from the above, is that if the difference between consumption and expenditures is considered to be important, and if one regards the already existing ways of dealing with this difference inadequate, not much insight can be gained from estimating a life cycle model in expenditure terms.[19] In order to be able to assess the importance of the difference between consumption and expenditures, it is necessary to collect consumption data next to expenditure data. Although it seems a difficult and expensive task to measure the consumption of households, the potential consequences of ignoring the difference between consumption and expenditures make research in this area more than necessary.

References.

Bar-Ilan, A., and S. Blinder (1988), "The life cycle permanent-income model and consumer durables", Annales d'Economie et de Statistique 9, pp. 71-91.

Blundell, R., and C. Meghir (1987), "Bivariate alternatives to the tobit model", Journal of Econometrics 34, pp. 179-200.

Deaton, A., and M. Irish (1984), "Statistical models for zero expenditures in household budgets", Journal of Public Economics 23, pp. 59-80.

Dunn, K. B., and K. J. Singleton (1986), "Modeling the term structure of interest rates under non-separable utility and durability of goods", Journal of Financial Economics 17, pp. 27-55.

Eichenbaum, M. S., L. P. Hansen, and K. J. Singleton (1988), "A time series analysis of representative agent models of consumption and leisure choice under uncertainty", Quarterly Journal of Economics CIII, pp. 51-78.

Hall, R. E. (1978), "Stochastic implications of the life cycle permanent income hypothesis: theory and evidence", Journal of Political Economy 86, pp. 971-988.

Hansen, L. P. and, K. J. Singleton (1983), "Stochastic consumption, risk aversion, and the temporal behavior of asset returns", Journal of Political Economy 91, pp. 249-265.

Hayashi, F. (1985), "The permanent income hypothesis and consumption durability: analysis based on Japanese panel data", Quarterly Journal of Economics C, pp. 1083-1113.

Hildenbrand, W. (1974), "Core and equilibria of a large economy", Princeton University Press, Princeton, New Jersey.

Meghir, C., and J. M. Robin (1989), " Frequency of purchase and the estimation of demand systems", University College London Discussion Paper in Economics 89-01.

Melenberg, B., and R. Alessie (1989), "A method to construct moments in the multi-good life cycle consumption model", FEW Research Memorandum 419, Tilburg University.

Muellbauer, J. (1988), "Habits, rationality and myopia in the life cycle consumption function", Annales d'Economie et de Statistique 9, pp. 47-70.

Neusser, K. (1988), "Intertemporal nonseparability, liquidity constraints, and seasonality of aggregate consumer expenditures: an empirical investigation", working paper.

Pashardes, P. (1986), "Myopic and forward looking behavior in a dynamic demand system", international Economic Review 27, pp. 387-397.

Pudney, S. (1989), "Modelling individual choice; the econometrics of corners, kinks and holes", Basil Blackwell, Oxford.

Spinneweyn, F. (1981), "Rational habit formation", European Economic Review 15, pp.91-109.

Appendix.

In order to formulate the second step model (4.4), it is necessary that c can be written as a function of e: $c = F(e)$. In this appendix, conditions guaranteeing the existence of such a function are given for the following two situations:

i) The solution to the first step model is unique.

In this case, strict concavity of $U(\cdot)$, and convexity of the choice set with respect to c are sufficient to guarantee the existence of the function $F(\cdot)$.[20] To demonstrate this, assume that $\bar{c}$ and $\hat{c}$ both are solutions of the first step model:

$$U(\bar{c}) = U(\hat{c}) = U^o = \max_c \{U(c); \Phi(e,c) \in Z, (c,e) \in C \times E\} \qquad (A.1)$$

Because of the convexity of the choice set with respect to c, the linear combination $\alpha\bar{c} + (1-\alpha)\hat{c}$ with $\alpha \in [0,1]$, is also a possible solution of model (A.1). The strict concavity of $U(\cdot)$ implies that $U(\alpha\bar{c} + (1-\alpha)\hat{c}) > \alpha U(\bar{c}) + (1-\alpha)U(\hat{c}) = U^o$. Since this implies that U^o is not optimal, the first step model must have an unique solution, say c^o. So, model (A.1) has an unique solution for each value of e, which is just another way of saying that c^o is a function of e: $c^o = F(e)$.

ii) The solution to the first step model is not unique.

In this situation, there is at each expenditure level more than one consumption level resulting in the optimum of the first step model. Hence, the optimal consumption is no longer a function of expenditure variables, but a correpondence. In, for example, Hildenbrand (1974) sufficient conditions are given under which one can still express c as a function of e. Because these conditions are rather technical, the reader is referred to Hildenbrand (1974), page 54 for further details.

Notes to chapter 4.

1 The fact that the infrequency of purchase model is a static approximation of this underlying dynamic process was already pointed out by Blundell and Meghir (1987).

2 To take account of this possible misspecification, some of the studies in this field (for example, Bludell and Meghir (1987), and Deaton and Irish (1984)) test the validity of the normality assumption. Alternatively, one could try to employ some semiparametric estimation procedure, thus making the normality assumption superfluous.

3 This assumption linking consumption to the available stock of a commodity is not always written down explicitly (see for example Pashardes (1986)). However, since consumers are usually assumed to derive utility (mainly) from consuming a good, and not from the fact that they possess a certain amount of it, such an assumption is necessary.

4 Choosing another decay structure, for example the form used by Dunn and Singleton (1986), does not change this result.

5 The aforementioned studies applying this 'lag approach', all use macro data. The only exception is Hayashi's study, in which quarterly household data are used. Given this lack of disaggregation, the subsequent discussion does have no bearing on the macro studies, and the relevance for Hayashi's work is limited.

6 Notice that the postponing of payments is not accounted for at all in the 'lag' model, since it links consumption only to past expenditures, not to future purchases.

7 Because habits refer to consumption, the lag polynomial linking consumption to expenditures does not represent habit formation, as claimed by, for example, Neusser (1988) and Muellbauer (1988).

8 Notice that there is another situation in which this will be the case, namely if the payments themselves are not delayed, but the reporting of them is. In most datasets, these two different mechanisms cannot be distinguished from each other.

9 An example of this situation could be the buying of clothes when the sales are on.

10 Notice that these links only deal with the physical quantities which are bought or consumed, not with the costs associated with these activities.

11 This is possible if there exists a (second-hand) market for each good in which any quantity can be sold at the same price per unit which holds if one buys the particular good for new. It will be assumed that such markets exist.

12 In the peak load pricing literature one sometimes comes across datasets containing information on the electricity use of households (see, for example, Bartels and Fiebig (1990)). This can be seen as an example in which the consumption of a good, electricity, is observed.

13 The upper bounds all are written as nonnegativity constraints to facilitate the application of the Lagrange multiplier rule.

14 The representation of the life cycle model as given in (4.3), is based upon the formulation used by Melenberg and Alessie (1989).

15 Remember that c is a vector of (consumption) functions which depend on the uncertainty inducing variables v. Hence, in order to determine the expected utility one needs the probability distribution of v. For an extensive discussion on the technical aspects of the model as given in (4.1), the reader is referred to Melenberg and Alessie (1989).

16 This is often the case in empirical work. In many studies information on national price indices must be used, since information on the prices consumers actually pay is lacking.

17 A much simpler version of model (3.1), which might be an interesting first step, results if one replaces the upper bounds on consumption by the following restriction: $\sum_{\tau=1}^{L} c_{\tau,i} \leq \sum_{\tau=1}^{L} e_{\tau,i}$, $i = 1,\ldots, M$. This restriction simply states that one's lifetime consumption of each good can not exceed the corresponding purchases made during this period.

18 It is possible, for instance, that testing on the basis of a subsystem leads to acception, whereas testing on the basis of the full system of restrictions would result in rejection of the model.

19 It is even possible that the outcomes of such a model which are considered by the researcher to be favourable, would not be obtained if the estimation was repeated using consumption data.

20 Notice that these two conditions often are imposed in studies of the life cycle model.

IN 1990 REEDS VERSCHENEN

435 Jack P.C. Kleijnen
Statistics and Deterministic Simulation Models: Why Not?

436 M.J.G. van Eijs, R.J.M. Heuts, J.P.C. Kleijnen
Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs

437 Jan A. Weststrate
Waiting times in a two-queue model with exhaustive and Bernoulli service

438 Alfons Daems
Typologie van non-profit organisaties

439 Drs. C.H. Veld en Drs. J. Grazell
Motieven voor de uitgifte van converteerbare obligatieleningen en warrantobligatieleningen

440 Jack P.C. Kleijnen
Sensitivity analysis of simulation experiments: regression analysis and statistical design

441 C.H. Veld en A.H.F. Verboven
De waardering van conversierechten van Nederlandse converteerbare obligaties

442 Drs. C.H. Veld en Drs. P.J.W. Duffhues
Verslaggevingsaspecten van aandelenwarrants

443 Jack P.C. Kleijnen and Ben Annink
Vector computers, Monte Carlo simulation, and regression analysis: an introduction

444 Alfons Daems
"Non-market failures": Imperfecties in de budgetsector

445 J.P.C. Blanc
The power-series algorithm applied to cyclic polling systems

446 L.W.G. Strijbosch and R.M.J. Heuts
Modelling (s,Q) inventory systems: parametric versus non-parametric approximations for the lead time demand distribution

447 Jack P.C. Kleijnen
Supercomputers for Monte Carlo simulation: cross-validation versus Rao's test in multivariate regression

448 Jack P.C. Kleijnen, Greet van Ham and Jan Rotmans
Techniques for sensitivity analysis of simulation models: a case study of the CO_2 greenhouse effect

449 Harrie A.A. Verbon and Marijn J.M. Verhoeven
Decision-making on pension schemes: expectation-formation under demographic change

450 Drs. W. Reijnders en Drs. P. Verstappen
Logistiek management marketinginstrument van de jaren negentig

451 Alfons J. Daems
Budgeting the non-profit organization
An agency theoretic approach

452 W.H. Haemers, D.G. Higman, S.A. Hobart
Strongly regular graphs induced by polarities of symmetric designs

453 M.J.G. van Eijs
Two notes on the joint replenishment problem under constant demand

454 B.B. van der Genugten
Iterated WLS using residuals for improved efficiency in the linear model with completely unknown heteroskedasticity

455 F.A. van der Duyn Schouten and S.G. Vanneste
Two Simple Control Policies for a Multicomponent Maintenance System

456 Geert J. Almekinders and Sylvester C.W. Eijffinger
Objectives and effectiveness of foreign exchange market intervention
A survey of the empirical literature

457 Saskia Oortwijn, Peter Borm, Hans Keiding and Stef Tijs
Extensions of the τ-value to NTU-games

458 Willem H. Haemers, Christopher Parker, Vera Pless and Vladimir D. Tonchev
A design and a code invariant under the simple group Co_3

459 J.P.C. Blanc
Performance evaluation of polling systems by means of the power-series algorithm

460 Leo W.G. Strijbosch, Arno G.M. van Doorne, Willem J. Selen
A simplified MOLP algorithm: The MOLP-S procedure

461 Arie Kapteyn and Aart de Zeeuw
Changing incentives for economic research in The Netherlands

462 W. Spanjers
Equilibrium with co-ordination and exchange institutions: A comment

463 Sylvester Eijffinger and Adrian van Rixtel
The Japanese financial system and monetary policy: A descriptive review

464 Hans Kremers and Dolf Talman
A new algorithm for the linear complementarity problem allowing for an arbitrary starting point

465 René van den Brink, Robert P. Gilles
A social power index for hierarchically structured populations of economic agents

IN 1991 REEDS VERSCHENEN

466 Prof.Dr. Th.C.M.J. van de Klundert - Prof.Dr. A.B.T.M. van Schaik
Economische groei in Nederland in een internationaal perspectief

467 Dr. Sylvester C.W. Eijffinger
The convergence of monetary policy - Germany and France as an example

468 E. Nijssen
Strategisch gedrag, planning en prestatie. Een inductieve studie binnen de computerbranche

469 Anne van den Nouweland, Peter Borm, Guillermo Owen and Stef Tijs
Cost allocation and communication

470 Drs. J. Grazell en Drs. C.H. Veld
Motieven voor de uitgifte van converteerbare obligatieleningen en warrant-obligatieleningen: een agency-theoretische benadering

471 P.C. van Batenburg, J. Kriens, W.M. Lammerts van Bueren and R.H. Veenstra
Audit Assurance Model and Bayesian Discovery Sampling

472 Marcel Kerkhofs
Identification and Estimation of Household Production Models

473 Robert P. Gilles, Guillermo Owen, René van den Brink
Games with Permission Structures: The Conjunctive Approach

474 Jack P.C. Kleijnen
Sensitivity Analysis of Simulation Experiments: Tutorial on Regression Analysis and Statistical Design

475 C.P.M. van Hoesel
An $O(nlogn)$ algorithm for the two-machine flow shop problem with controllable machine speeds

476 Stephan G. Vanneste
A Markov Model for Opportunity Maintenance

477 F.A. van der Duyn Schouten, M.J.G. van Eijs, R.M.J. Heuts
Coordinated replenishment systems with discount opportunities

478 A. van den Nouweland, J. Potters, S. Tijs and J. Zarzuelo
Cores and related solution concepts for multi-choice games

479 Drs. C.H. Veld
Warrant pricing: a review of theoretical and empirical research

480 E. Nijssen
De Miles and Snow-typologie: Een explorative studie in de meubelbranche

481 Harry G. Barkema
Are managers indeed motivated by their bonuses?

513 Drs. J. Dagevos, Drs. L. Oerlemans, Dr. F. Boekema
Regional economic policy, economic technological innovation and networks

514 Erwin van der Krabben
Het functioneren van stedelijke onroerend-goed-markten in Nederland - een theoretisch kader

515 Drs. E. Schaling
European central bank independence and inflation persistence

516 Peter M. Kort
Optimal abatement policies within a stochastic dynamic model of the firm